METRIC SPACES: FINAL EXAM 2013

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Evaluation: $\min\Big(100\%, \max\big(5\,\text{prb} \times 20\% \cdot \left[\begin{smallmatrix}1.00\\1.15^{\text{top}}\end{smallmatrix}\right],\ \sum_{i=1}^{6} \text{h/w} \times 5\% + 5\,\text{prb} \times 14\% \cdot \left[\begin{smallmatrix}1.00\\1.15^{\text{top}}\end{smallmatrix}\right]\big)\Big)$.

Problem 1. Let $(\mathcal{X}, \mathrm{d})$ be a non-empty metric space, r and s be two positive radii, and $B_r^{\mathrm{d}}(x) = B_s^{\mathrm{d}}(y)$ for some $x, y \in \mathcal{X}$.

- Is it true that $r = s$?
- Is it true that $x = y$?

Problem 2 (top). Let $A \subseteq \mathcal{X}$ be a subset of a non-empty space $\mathcal{X}$. Prove that the boundary ∂A of A is closed in $\mathcal{X}$.

Problem 3 (top). Let $A \subseteq \mathcal{X}$ be a connected subset of a space $\mathcal{X}$ and suppose that $A \subseteq B \subseteq \overline{A}$. Is the subset $B \subseteq \mathcal{X}$ always connected?

(state and prove)

Problem 4 (top). Suppose that a non-empty Hausdorff space $\mathcal{X}$ is compact and there is a continuous map $f\colon \mathcal{X} \to \mathcal{X}$. Let $\mathcal{X}_1 = \mathcal{X}$ and put $\mathcal{X}_{n+1} = f(\mathcal{X}_n)$ inductively for all $n \in \mathbb{N}$.

- Prove that $A = \bigcap_{n=1}^{+\infty} \mathcal{X}_n$ is non-empty.

Problem 5. Prove that every discrete metric space $(\mathcal{X}, \mathrm{d}_0)$ is complete.

Date: April 4, 2013.
Do not postpone your success until July 5, 2013. GOOD LUCK!